

# Energy level displacement of excited $np$ states of kaonic hydrogen

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## Abstract

We compute the energy level displacement of the excited  $np$  states of kaonic hydrogen within the quantum field theoretic and relativistic covariant model of strong low-energy  $\bar{K}N$  interactions suggested in EPJA **21**, 11 (2004). For the width of the energy level of the excited  $2p$  state of kaonic hydrogen, caused by strong low-energy interactions, we find  $\Gamma_{2p} = 2 \text{ meV} = 3 \times 10^{12} \text{ sec}^{-1}$ . This result is important for the theoretical analysis of the  $X$ -ray yields in kaonic hydrogen.

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# 1 Introduction

Recently [1] we have computed the energy level displacement of the ground state of kaonic hydrogen

$$-\epsilon_{1s}^{(\text{th})} + i \frac{\Gamma_{1s}^{(\text{th})}}{2} = (-203 \pm 15) + i(113 \pm 14) \text{ eV}. \quad (1.1)$$

This result has been obtained within a quantum field theoretic and relativistic covariant model of strong low-energy  $\bar{K}N$  interactions near threshold of  $K^-p$  scattering, based on the dominant role of strange resonances  $\Lambda(1405)$  and  $\Sigma(1750)$  in the s-channel of low-energy elastic and inelastic  $K^-p$  scattering and the exotic four-quark ( or  $K\bar{K}$  molecules) scalar states  $a_0(980)$  and  $f_0(980)$  in the  $t$ -channel of low-energy elastic  $K^-p$  scattering.

The theoretical result (1.1) agrees well with recent experimental data obtained by the DEAR Collaboration [2]:

$$-\epsilon_{1s}^{(\text{exp})} + i \frac{\Gamma_{1s}^{(\text{exp})}}{2} = (-194 \pm 41) + i(125 \pm 59) \text{ eV}. \quad (1.2)$$

A systematic analysis of corrections, caused by electromagnetic and QCD isospin-breaking interactions, to the energy level displacements of the  $ns$  states of kaonic hydrogen, where  $n$  is the principal quantum number, has been recently carried out by Meißner, Raha and Rusetsky [3] within Effective Field Theory by using the non-relativistic effective Lagrangian approach based on Chiral Perturbation Theory (ChPT) by Gasser and Leutwyler [4, 5]. For the S-wave amplitude of  $K^-N$  scattering near threshold, computed in [1, 6], the energy level displacement of the ground state of kaonic hydrogen obtained by Meißner *et al.* [3] is equal to

$$-\epsilon_{1s}^{(\text{th})} + i \frac{\Gamma_{1s}^{(\text{th})}}{2} = (-266 \pm 23) + i(177 \pm 21) \text{ eV}.$$

This agrees well with both our theoretical result (1.1) and experimental data (1.2) within 1.5 standard deviations for the shift and one standard deviation for the width.

In this paper we compute the energy level displacement of the excited  $np$  states of kaonic hydrogen, where  $n$  is the principal quantum number and  $p$  corresponds to the excited state with  $\ell = 1$ . The knowledge of the energy level displacement of the excited  $np$  states of kaonic hydrogen is very important for the understanding of the accuracy of experimental measurements of the energy level displacement of the ground state of kaonic hydrogen and the theoretical analysis of the X-ray yields in kaonic hydrogen [2], [7]–[13].

The paper is organized as follows. In Section 2 we extend our approach to the description of low-energy  $K^-p$  interaction in the S-wave state to the analysis of the low-energy  $K^-p$  interaction in the P-wave state with a total angular momentum  $J = 3/2$  and  $J = 1/2$ , respectively. We compute the P-wave scattering lengths of elastic  $K^-p$  scattering and the energy level shift of the  $np$  excited state of kaonic hydrogen. In Section 3 we compute the P-wave scattering lengths of inelastic reactions  $K^-p \rightarrow Y\pi$ , where  $Y\pi = \Sigma^-\pi^+, \Sigma^+\pi^-, \Sigma^0\pi^0$  and  $\Lambda^0\pi^0$ . We compute the energy level width of the  $np$  excited state of kaonic hydrogen. For the  $2p$  state of kaonic hydrogen we get  $\Gamma_{2p} = 2 \text{ meV} = 3 \times 10^{12} \text{ sec}^{-1}$ . The rate of the hadronic decays of kaonic hydrogen from the  $np$  excited state is important

for the theoretical analysis of the  $X$ -ray yields in kaonic hydrogen, which are the main experimental tool for the measurement of the energy level displacement of the ground state of kaonic hydrogen [2]. In the Conclusion we discuss the obtained results.

## 2 Energy level displacement of the $n\ell$ excited states of kaonic hydrogen. General formulas

According to [14], the energy level displacement of the excited  $n\ell$  states of kaonic hydrogen can be defined by

$$-\epsilon_{n\ell} + i \frac{\Gamma_{n\ell}}{2} = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} \int \frac{d^3k}{(2\pi)^3} \frac{\Phi_{n\ell}^{\dagger}(k)}{\sqrt{2E_{K^-}(k)2E_p(k)}} \int \frac{d^3q}{(2\pi)^3} \frac{\Phi_{n\ell}(q)}{\sqrt{2E_{K^-}(q)2E_p(q)}} \\ \times \iint \frac{d\Omega_{\vec{k}}}{\sqrt{4\pi}} \frac{d\Omega_{\vec{q}}}{\sqrt{4\pi}} Y_{\ell m}^*(\vartheta_{\vec{k}}, \varphi_{\vec{k}}) M(K^-(\vec{q})p(-\vec{q}, \sigma_p) \rightarrow K^-(\vec{k})p(-\vec{k}, \sigma_p)) Y_{\ell m}(\vartheta_{\vec{q}}, \varphi_{\vec{q}}), \quad (2.1)$$

where  $M(K^-(\vec{q})p(-\vec{q}, \sigma_p) \rightarrow K^-(\vec{k})p(-\vec{k}, \sigma_p))$  is the amplitude of elastic  $K^-p$  scattering,  $\Phi_{n\ell}(k)$  is a radial wave function of kaonic hydrogen in the  $n\ell$  excited state in momentum representation. It is defined by [14]

$$\Phi_{n\ell}(k) = \sqrt{4\pi} \int_0^{\infty} j_{\ell}(kr) R_{n\ell}(r) r^2 dr, \quad (2.2)$$

where  $j_{\ell}(kr)$  are spherical Bessel functions [15] and  $R_{n\ell}(r)$  is a radial wave function of kaonic hydrogen in the coordinate representation [16]:

$$R_{n\ell}(r) = -\frac{2}{n^2} \sqrt{\frac{(n-\ell-1)!}{[(n+\ell)!]^3 a_B^3}} \left(\frac{2}{n} \frac{r}{a_B}\right)^{\ell} e^{-r/na_B} L_{n+\ell}^{2\ell+1}\left(\frac{2}{n} \frac{r}{a_B}\right). \quad (2.3)$$

Here  $L_{n+\ell}^{2\ell+1}(\rho)$  are the generalised Laguerre polynomials given by [16]

$$L_{n+\ell}^{2\ell+1}(\rho) = (-1)^{2\ell+1} \frac{(n+\ell)!}{(n-\ell-1)!} \rho^{-(2\ell+1)} e^{\rho} \frac{d^{n-\ell-1}}{d\rho^{n-\ell-1}} (\rho^{n+\ell} e^{-\rho}), \quad (2.4)$$

where  $\rho = r/na_B$  and  $a_B = 1/\alpha\mu = 83 \text{ fm}$  is the Bohr radius of kaonic hydrogen with  $\mu = m_K m_N / (m_K + m_N) = 324 \text{ MeV}$  and  $\alpha = 1/137.036$  are the reduced mass of the  $K^-p$  pair, computed for  $m_K = 494 \text{ MeV}$  and  $m_N = 940 \text{ MeV}$ , and the fine-structure constant, respectively. Spherical harmonics  $Y_{\ell m}(\vartheta, \varphi)$  are normalized by

$$\int d\Omega Y_{\ell' m'}^*(\vartheta, \varphi) Y_{\ell m}(\vartheta, \varphi) = \delta_{\ell' \ell} \delta_{m' m}, \quad (2.5)$$

where  $d\Omega = \sin \vartheta d\vartheta d\varphi$  is a volume element of solid angle.

In Eq.(2.1) due to the wave functions  $\Phi_{n\ell}^{\dagger}(k)$  and  $\Phi_{n\ell}(q)$  the integrand of the momentum integrals is concentrated at momenta of order of  $k \sim q \sim 1/na_B = \alpha\mu/n = 2.4/n \text{ MeV}$ . Therefore, the amplitude of elastic  $K^-p$  scattering can be defined in the low-energy limit at  $k, q \rightarrow 0$ . Since in the low-energy limit there is no spin-flip in the

transition  $K^- + p \rightarrow K^- + p$  the amplitude of low-energy elastic  $K^-p$  scattering can be determined by [17]–[19] (see also [14]):

$$\begin{aligned}
M(K^-(\vec{q})p(-\vec{q}, \sigma_p) \rightarrow K^-(\vec{k})p(-\vec{k}, \sigma_p)) &= 8\pi \sqrt{s} \sum_{\ell'=0}^{\infty} [(\ell' + 1) f_{\ell'+}(\sqrt{kq}) + \ell' f_{\ell'-}(\sqrt{kq})] \\
&\times P_{\ell'}(\cos \vartheta) = 8\pi \sqrt{s} \sum_{\ell'=0}^{\infty} [(\ell' + 1) f_{\ell'+}(\sqrt{kq}) + \ell' f_{\ell'-}(\sqrt{kq})] \sum_{m'=-\ell'}^{\ell'} \frac{4\pi}{2\ell' + 1} Y_{\ell'm'}^*(\vartheta_{\vec{q}}, \varphi_{\vec{q}}) \\
&\times Y_{\ell'm'}(\vartheta_{\vec{k}}, \varphi_{\vec{k}}), \tag{2.6}
\end{aligned}$$

where  $\sqrt{s}$  is the total energy in the  $s$ -channel of  $K^-p$  scattering,  $P_{\ell'}(\cos \vartheta)$  are Legendre polynomials [15] and  $\vartheta$  is the angle between the relative momenta  $\vec{k}$  and  $\vec{q}$ . The amplitudes  $f_{\ell'+}(\sqrt{kq})$  and  $f_{\ell'-}(\sqrt{kq})$  describe elastic  $K^-p$  scattering in the states with a total angular momentum  $J = \ell' + 1/2$  and  $J = \ell' - 1/2$ , respectively. They are defined by

$$\begin{aligned}
f_{\ell'+}(\sqrt{kq}) &= \frac{1}{2i\sqrt{kq}} \left( \eta_{\ell'+}(\sqrt{kq}) e^{+2i\delta_{\ell'+}(\sqrt{kq})} - 1 \right), \\
f_{\ell'-}(\sqrt{kq}) &= \frac{1}{2i\sqrt{kq}} \left( \eta_{\ell'-}(\sqrt{kq}) e^{+2i\delta_{\ell'-}(\sqrt{kq})} - 1 \right), \tag{2.7}
\end{aligned}$$

where  $\eta_{\ell'\pm}(\sqrt{kq})$  and  $\delta_{\ell'\pm}(\sqrt{kq})$  are inelasticities and phase shifts of elastic  $K^-p$  scattering [17]–[19].

Near threshold the amplitudes  $f_{\ell'+}(\sqrt{kq})$  and  $f_{\ell'-}(\sqrt{kq})$  possess the real and imaginary parts. The real parts of the amplitudes  $f_{\ell'+}(\sqrt{kq})$  and  $f_{\ell'-}(\sqrt{kq})$  are defined by the  $\ell'$ -wave scattering lengths of  $K^-p$  scattering [18, 14]

$$\begin{aligned}
\mathcal{R}e f_{\ell'+}(\sqrt{kq}) &= a_{\ell'+}^{K^-p}(kq)^{\ell'}, \\
\mathcal{R}e f_{\ell'-}(\sqrt{kq}) &= a_{\ell'-}^{K^-p}(kq)^{\ell'}. \tag{2.8}
\end{aligned}$$

Using (2.8) for the shift of the energy level of the  $n\ell$  excited state of kaonic hydrogen we obtain [14]

$$\epsilon_{n\ell} = -\frac{2\pi}{\mu} \frac{(\ell + 1) a_{\ell+}^{K^-p} + \ell a_{\ell-}^{K^-p}}{2\ell + 1} \left| \int \frac{d^3k}{(2\pi)^3} \sqrt{\frac{m_K m_N}{E_{K^-}(k) E_p(k)}} k^\ell \Phi_{n\ell}(k) \right|^2. \tag{2.9}$$

The imaginary parts of the amplitudes  $f_{\ell'+}(\sqrt{kq})$  and  $f_{\ell'-}(\sqrt{kq})$  are defined by inelastic channels  $K^-p \rightarrow \Sigma^- \pi^+$ ,  $K^-p \rightarrow \Sigma^+ \pi^-$ ,  $K^-p \rightarrow \Sigma^0 \pi^0$  and  $K^-p \rightarrow \Lambda^0 \pi^0$ . According to [14], the width  $\Gamma_{n\ell}$  of the energy level of the  $n\ell$  excited state of kaonic hydrogen is given by

$$\begin{aligned}
\Gamma_{n\ell} &= \frac{4\pi}{\mu} \sum_{Y\pi} \left[ \frac{(\ell + 1) a_{\ell+}^{Y\pi} + \ell a_{\ell-}^{Y\pi}}{2\ell + 1} \right]^2 [k_{Y\pi}(W_{n\ell})]^{2\ell+1} \\
&\times \left| \int \frac{d^3k}{(2\pi)^3} \sqrt{\frac{m_K m_N}{E_{K^-}(k) E_p(k)}} k^\ell \Phi_{n\ell}(k) \right|^2, \tag{2.10}
\end{aligned}$$

where we sum over all  $Y\pi$  pairs  $Y\pi = \Sigma^+ \pi^-, \Sigma^- \pi^+, \Sigma^0 \pi^0$  and  $\Lambda^0 \pi^0$ ;  $k_{Y\pi}(W_{n\ell})$  is a relative momentum of the  $Y\pi$  pair

$$k_{Y\pi}(W_{n\ell}) = \frac{\sqrt{(W_{n\ell}^2 - (m_Y + m_\pi)^2)(W_{n\ell}^2 - (m_Y - m_\pi)^2)}}{2W_{n\ell}} \tag{2.11}$$

with  $W_{n\ell} = m_K + m_N + E_{n\ell}$  and  $E_{n\ell}$  is the binding energy of kaonic hydrogen in the  $n\ell$  excited state [19].

The analysis of experimental data obtained by the DEAR Collaboration [2] requires the knowledge of the energy level displacement of the excited  $np$  states. For  $\ell = 1$  the formulas (2.9) and (2.10) read

$$\begin{aligned}\epsilon_{np} &= -\frac{2\pi}{3} \frac{1}{\mu} (2a_{3/2}^{K^-p} + a_{1/2}^{K^-p}) \left| \int \frac{d^3k}{(2\pi)^3} \sqrt{\frac{m_K m_N}{E_{K^-}(k) E_p(k)}} k \Phi_{np}(k) \right|^2, \\ \Gamma_{np} &= \frac{4\pi}{9} \frac{1}{\mu} \sum_{Y\pi} (2a_{3/2}^{Y\pi} + a_{1/2}^{Y\pi})^2 k_{Y\pi}^3 \left| \int \frac{d^3k}{(2\pi)^3} \sqrt{\frac{m_K m_N}{E_{K^-}(k) E_p(k)}} k \Phi_{np}(k) \right|^2,\end{aligned}\quad (2.12)$$

where  $\Phi_{np}(k)$  is the radial wave function of kaonic hydrogen in the  $np$  excited state in the momentum representation, the indices 3/2 and 1/2 denote the P-wave amplitudes of the reactions  $K^-p \rightarrow K^-p$  and  $K^-p \rightarrow Y\pi$  with total angular momentum  $J = 3/2$  and  $J = 1/2$ , respectively [17]–[19].

The momentum integral in the r.h.s. of (2.12) has been computed in [14]. Using this result the energy level displacement of the  $np$  excited states reads

$$\begin{aligned}\epsilon_{np} &= -\frac{2}{3} \frac{\alpha^5}{n^3} \left(1 - \frac{1}{n^2}\right) \left(\frac{m_K m_N}{m_K + m_N}\right)^4 (2a_{3/2}^{K^-p} + a_{1/2}^{K^-p}), \\ \Gamma_{np} &= \frac{4}{9} \frac{\alpha^5}{n^3} \left(1 - \frac{1}{n^2}\right) \left(\frac{m_K m_N}{m_K + m_N}\right)^4 \sum_{Y\pi} (2a_{3/2}^{Y\pi} + a_{1/2}^{Y\pi})^2 k_{Y\pi}^3.\end{aligned}\quad (2.13)$$

Thus, the problem of the calculation of the energy level displacement of the  $np$  excited states of kaonic hydrogen reduces to the problem of the calculation of the P-wave scattering lengths  $a_{1/2}^{K^-p}$  and  $a_{3/2}^{K^-p}$  of elastic  $K^-p$  scattering and P-wave scattering lengths  $a_{1/2}^{Y\pi}$  and  $a_{3/2}^{Y\pi}$  of inelastic reactions  $K^-p \rightarrow Y\pi$  with  $Y\pi = \Sigma^+\pi^-, \Sigma^-\pi^+, \Sigma^0\pi^0$  and  $\Lambda^0\pi^0$ .

### 3 Model for low-energy $K^-p$ scattering in the P-wave state

For the description of the P-wave amplitude of low-energy  $K^-p$  scattering we follow [1, 6] and assume that

- (i) the amplitudes with total angular momentum  $J = 1/2$  are defined by the contributions of the elastic background and the octets of baryon resonances with spin 1/2 and positive parity such as  $(N(1440), \Lambda^0(1600), \Sigma(1660)) = B_1(\mathbf{8})$  and  $(N(1710), \Lambda^0(1810), \Sigma(1880)) = B_2(\mathbf{8})$  and
- (ii) the amplitudes with total angular momentum  $J = 3/2$  are defined by the contributions of the elastic background and the baryon resonances with spin 3/2 and positive parity from decuplet  $(\Delta(1232), \Sigma(1385)) = B_3(\mathbf{10})$  and octet  $(N(1720), \Lambda^0(1890), \Sigma(1840)) = B_4(\mathbf{8})$  [20]. We would like to emphasize that the baryon resonances we will treat as elementary particles defined by local fields and local phenomenological Lagrangians with phenomenological coupling constants [1, 6] (see also [21]). The contribution of the octet of low-lying baryons with spin 1/2 and positive parity  $(N(940), \Lambda^0(1116), \Sigma(1193)) = B(\mathbf{8})$  we include to the elastic background.

### 3.1 P-wave scattering lengths of elastic $K^-p$ scattering

The P-wave amplitude of elastic  $K^-p$  scattering at threshold is defined by two P-wave scattering lengths  $a_{1/2}^{K^-p}$  and  $a_{3/2}^{K^-p}$  caused by the interactions of the  $K^-p$  pair in the states with a total angular momentum  $J = 1/2$  and  $J = 3/2$ , respectively.

#### P-wave scattering length $a_{1/2}^{K^-p}$

According to our approach to the description of the low-energy  $K^-p$  interaction in the S-wave state extended to the low-energy  $K^-p$  interaction in the P-wave state, the amplitude  $a_{1/2}^{K^-p}$  has the following form

$$a_{1/2}^{K^-p} = (a_{1/2}^{K^-p})_B + \sum_R (a_{1/2}^{K^-p})_R, \quad (3.1)$$

where  $(a_{1/2}^{K^-p})_B$  is the contribution of an elastic background and  $(a_{1/2}^{K^-p})_R$  is the contribution of the baryon resonance  $R = \Lambda_1^0, \Sigma_1^0, \Lambda_2^0$  and  $\Sigma_2^0$ .

#### Resonance contribution to P-wave scattering length $a_{1/2}^{K^-p}$

The phenomenological low-energy interactions  $B_1(\mathbf{8})B(\mathbf{8})P(\mathbf{8})$  and  $B_2(\mathbf{8})B(\mathbf{8})P(\mathbf{8})$ , necessary for the calculation of the contribution of the baryon resonances to the P-wave amplitude  $a_{1/2}^{K^-p}$ , can be defined using the results obtained in [6]. As a result for the sum of the baryon resonance contributions we obtain

$$\begin{aligned} \sum_R (a_{1/2}^{K^-p})_R = & -\frac{1}{8\pi} \frac{1}{m_K + m_N} \left[ \frac{1}{\sqrt{3}} (3 - 2\alpha_1) g_{\pi NN_1} \right]^2 \frac{1}{2m_N} \frac{1}{m_{\Lambda_1^0} - m_N - m_K} \\ & -\frac{1}{8\pi} \frac{1}{m_K + m_N} \left[ (2\alpha_1 - 1) g_{\pi NN_1} \right]^2 \frac{1}{2m_N} \frac{1}{m_{\Sigma_1^0} - m_N - m_K} \\ & -\frac{1}{8\pi} \frac{1}{m_K + m_N} \left[ \frac{1}{\sqrt{3}} (3 - 2\alpha_2) g_{\pi NN_2} \right]^2 \frac{1}{2m_N} \frac{1}{m_{\Lambda_2^0} - m_N - m_K} \\ & -\frac{1}{8\pi} \frac{1}{m_K + m_N} \left[ (2\alpha_2 - 1) g_{\pi NN_2} \right]^2 \frac{1}{2m_N} \frac{1}{m_{\Sigma_2^0} - m_N - m_K}. \end{aligned} \quad (3.2)$$

The coupling constants of the interactions  $K^-pB_1(\mathbf{8})$  and  $K^-pB_2(\mathbf{8})$  are equal to:  $g_{\pi NN_1} = 6.28$ ,  $\alpha_1 = 0.85$ ,  $g_{\pi NN_2} = 1.20$  and  $\alpha_2 = -1.55$ . These numerical values of the coupling constants one can obtain by using the phenomenological  $SU(3)$ -invariant interactions  $B_1(\mathbf{8})B(\mathbf{8})P(\mathbf{8})$  and  $B_2(\mathbf{8})B(\mathbf{8})P(\mathbf{8})$  (see [6]), where  $P(\mathbf{8})$  is the octet of low-lying pseudoscalar mesons, and experimental data on the partial widths of the resonances  $B_1(\mathbf{8})$  and  $B_2(\mathbf{8})$  [20]. Using the recommended masses for the resonances  $m_{\Lambda_1^0} = 1600$  MeV,  $m_{\Sigma_1} = 1660$  MeV,  $m_{\Lambda_2^0} = 1810$  MeV and  $m_{\Sigma_2} = 1880$  MeV we compute

$$\sum_R (a_{1/2}^{K^-p})_R = -0.013 m_\pi^{-3}. \quad (3.3)$$

Now we proceed to computing the contribution of the elastic background  $(a_{1/2}^{K^-p})_B$ .

#### Elastic background contribution to the P-wave scattering length $a_{1/2}^{K^-p}$

According to [1], the contribution of the elastic background  $(a_{1/2}^{K^-p})_B$  to the P-wave scattering length  $a_{1/2}^{K^-p}$  should be defined by the contribution of all low-energy interactions  $(a_{1/2}^{K^-p})_{CA}$ , which can be described within the Effective Chiral Lagrangian (the ECL) approach [22] or that is equivalent within Current Algebra (CA) [23]–[30], supplemented by soft-kaon theorems (SKT) [26]–[30], and the contribution  $(a_{1/2}^{K^-p})_{K\bar{K}}$  of low-energy exchanges with the exotic scalar mesons  $a_0(980)$  and  $f_0(980)$ , which are four-quark states [31, 32] or  $K\bar{K}$  molecules [32, 33]. The description of strong low-energy interactions of these mesons goes beyond the ECL approach, describing strong low-energy interactions of mesons with  $q\bar{q}$  and baryons with  $qqq$  quark structures. Recent experimental confirmation of the exotic structure of the scalar mesons  $a_0(980)$  and  $f_0(980)$  has been obtained by the DEAR Collaboration at DAPHNE [34].

Thus, the P-wave scattering length  $(a_{1/2}^{K^-p})_B$  is defined by

$$(a_{1/2}^{K^-p})_B = (a_{1/2}^{K^-p})_{CA} + (a_{1/2}^{K^-p})_{K\bar{K}}. \quad (3.4)$$

Using the results obtained in [1] we compute the contribution of the exotic scalar mesons

$$a_{1/2,K\bar{K}}^{K^-p} = -\frac{1}{\pi} \frac{m_N}{m_K + m_N} \frac{g_D g_0}{m_{a_0}^4} \left(1 - \frac{1}{8} \frac{m_{a_0}^2}{m_N^2}\right) = -0.018 \text{ m}_\pi^{-3}, \quad (3.5)$$

where  $m_{f_0} = m_{a_0} = 980 \text{ MeV}$ ,  $g_0 = g_{a_0 K^+ K^-} = g_{f_0 K^+ K^-} = 2746 \text{ MeV}$  [32] and  $g_D = \xi g_{\pi NN}/g_A = 0.95 g_{\pi NN}$ . For the calculation of  $g_D$  we have used  $\xi = 1.2$  [1] and  $g_A = 1.267$  [20]. The coupling constant  $g_{\pi NN}$  of the  $\pi NN$  interaction is equal to  $g_{\pi NN} = 13.21$  [35] (see also [36] by Ericson, Loiseau and Wycech, where the authors have obtained  $g_{\pi NN} = 13.28 \pm 0.08$ ).

The contribution to the P-wave amplitude, caused by the ECL interactions, we represent in the form of the superposition of the contributions of the  $\Lambda^0(1116)$  and  $\Sigma^0(1193)$  hyperon exchanges and the term  $(a_{1/2}^{K^-p})_{SKT}$ , which can be computed applying the soft-kaon technique [26]–[30]. Thus, we get

$$\begin{aligned} (a_{1/2}^{K^-p})_{CA} &= (a_{1/2}^{K^-p})_{SKT} \\ &- \frac{1}{8\pi} \frac{1}{m_K + m_N} \left[ \frac{1}{\sqrt{3}} (3 - 2\alpha) g_{\pi NN} \right]^2 \frac{1}{2m_N} \frac{1}{m_{\Lambda^0} - m_N - m_K} \\ &- \frac{1}{8\pi} \frac{1}{m_K + m_N} \left[ (2\alpha - 1) g_{\pi NN} \right]^2 \frac{1}{2m_N} \frac{1}{m_{\Sigma^0} - m_N + m_K}. \end{aligned} \quad (3.6)$$

For  $m_{\Lambda^0} = 1116 \text{ MeV}$ ,  $m_{\Sigma^0} = 1193 \text{ MeV}$ ,  $g_{\pi NN} = 13.21$  and  $\alpha = 0.64$  [6] we obtain

$$(a_{1/2}^{K^-p})_{CA} = (a_{1/2}^{K^-p})_{SKT} + 0.024 \text{ m}_\pi^{-3}. \quad (3.7)$$

Summing up the contributions, for the P-wave scattering length  $a_{1/2}^{K^-p}$  of  $K^-p$  scattering with a total angular momentum  $J = 1/2$  we get

$$a_{1/2}^{K^-p} = (a_{1/2}^{K^-p})_{SKT} - 0.007 \text{ m}_\pi^{-3}. \quad (3.8)$$

We suggest to compute the quantity  $(a_{1/2}^{K^-p})_{SKT}$  together with  $(a_{3/2}^{K^-p})_{SKT}$ , the contribution of the elastic background to the P-wave scattering length  $a_{3/2}^{K^-p}$  of  $K^-p$  scattering with a total angular momentum  $J = 3/2$ .

### P-wave scattering length $a_{3/2}^{K^-p}$

The P-wave scattering length  $a_{3/2}^{K^-p}$  we represent by

$$a_{3/2}^{K^-p} = (a_{3/2}^{K^-p})_B + \sum_R (a_{3/2}^{K^-p})_R, \quad (3.9)$$

where  $(a_{3/2}^{K^-p})_B$  is the contribution of an elastic background and  $(a_{3/2}^{K^-p})_R$  is the contribution of the baryon resonances  $R = \Sigma_3^0, \Lambda_4^0$  and  $\Sigma_4^0$ . The elastic background  $(a_{3/2}^{K^-p})_B$  does not contain rapidly changing contributions, therefore below we assume that  $(a_{3/2}^{K^-p})_B = (a_{3/2}^{K^-p})_{SKT}$ .

### Resonance contribution to the P-wave scattering length $a_{3/2}^{K^-p}$

The phenomenological low-energy interaction of the resonance  $\Sigma_3^0$  with octets low-lying baryons  $B(\mathbf{8})$  and pseudoscalar mesons  $P(\mathbf{8})$  is defined by [18, 19, 37] (see also [20]):

$$\begin{aligned} \mathcal{L}_{\Sigma_3^0 BP}(x) &= \\ &= \frac{g_{\pi NN}}{\sqrt{6}m_N} \bar{\Sigma}_{3\mu}^0(x) [\Sigma^+(x) \partial^\mu \pi^-(x) - \Sigma^-(x) \partial^\mu \pi^+(x) + p(x) \partial^\mu K^-(x) + \sqrt{3} \Lambda^0(x) \partial^\mu \pi^0(x)] \\ &+ \frac{g_{\pi NN}}{\sqrt{6}m_N} [\bar{\Sigma}^+(x) \partial^\mu \pi^+(x) - \bar{\Sigma}^-(x) \partial^\mu \pi^-(x) - \bar{p}(x) \partial^\mu K^+(x) + \sqrt{3} \bar{\Lambda}^0(x) \partial^\mu \pi^0(x)] \Sigma_{3\mu}^0(x), \end{aligned} \quad (3.10)$$

where we have written down only those interactions which contribution to the P-wave amplitude of low-energy  $K^-p$  scattering.

Using (3.10) we compute the contribution of the resonance  $\Sigma(1385)$  to the P-wave scattering length  $a_{3/2}^{K^-p}$ :

$$\begin{aligned} (a_{3/2}^{K^-p})_{\Sigma_3^0} &= -\frac{g_{\pi NN}^2}{36\pi m_N} \frac{1}{m_K + m_N} \frac{m_{\Sigma_3^0}}{m_{\Sigma_3^0}^2 - (m_K + m_N)^2} \left\{ \left[ 1 - \frac{1}{2} \frac{m_K}{m_N} - \frac{1}{4} \frac{m_K^2}{m_N^2} \right] \right. \\ &+ \frac{(m_K + m_N)}{m_{\Sigma_3^0}} \left[ 1 + \frac{1}{2} \frac{m_K}{m_N} \frac{(m_K + m_N)}{m_{\Sigma_3^0}} - \frac{1}{4} \frac{m_K^2}{m_N^2} \left( 1 + \frac{(m_K + m_N)}{m_{\Sigma_3^0}} - \frac{(m_K + m_N)^2}{m_{\Sigma_3^0}^2} \right) \right] \Big\} = \\ &= 0.060 m_\pi^{-3}. \end{aligned} \quad (3.11)$$

The contribution of the resonances  $\Lambda_4^0$  and  $\Sigma_4^0$  to  $a_{3/2}^{K^-p}$  is equal to

$$\begin{aligned} \sum_{R=\Lambda_4^0, \Sigma_4^0} (a_{3/2}^{K^-p})_R &= -\frac{1}{6\pi m_N} \left[ \frac{1}{\sqrt{3}} (3 - 2\alpha_4) g_{\pi NN_4} \right]^2 \frac{1}{m_K + m_N} \frac{m_{\Lambda_4^0}}{m_{\Lambda_4^0}^2 - (m_K + m_N)^2} \\ &\times \left\{ \left[ 1 - \frac{1}{2} \frac{m_K}{m_N} - \frac{1}{4} \frac{m_K^2}{m_N^2} \right] + \frac{(m_K + m_N)}{m_{\Lambda_4^0}} \left[ 1 + \frac{1}{2} \frac{m_K}{m_N} \frac{(m_K + m_N)}{m_{\Lambda_4^0}} \right] \right\} \end{aligned}$$



$$\begin{aligned}
& -\frac{1}{4} \frac{m_K^2}{m_N^2} \left( 1 + \frac{(m_K + m_N)}{m_{\Lambda_4^0}} - \frac{(m_K + m_N)^2}{m_{\Lambda_4^0}^2} \right) \Big] \Big\} \\
& -\frac{1}{6\pi m_N} \left[ (2\alpha_4 - 1)g_{\pi NN_4} \right]^2 \frac{1}{m_K + m_N} \frac{m_{\Sigma_4^0}}{m_{\Sigma_4^0}^2 - (m_K + m_N)^2} \\
& \times \left\{ \left[ 1 - \frac{1}{2} \frac{m_K}{m_N} - \frac{1}{4} \frac{m_K^2}{m_N^2} \right] + \frac{(m_K + m_N)}{m_{\Sigma_4^0}} \left[ 1 + \frac{1}{2} \frac{m_K}{m_N} \frac{(m_K + m_N)}{m_{\Sigma_4^0}} \right. \right. \\
& \left. \left. - \frac{1}{4} \frac{m_K^2}{m_N^2} \left( 1 + \frac{(m_K + m_N)}{m_{\Sigma_4^0}} - \frac{(m_K + m_N)^2}{m_{\Sigma_4^0}^2} \right) \right] \right\}. \tag{3.12}
\end{aligned}$$

Using the experimental data on the resonances from the octet  $B_4(8)$  [20] we compute the coupling constants  $g_{\pi NN_4} = 1.16$  and  $\alpha_4 = 0.32$ . For  $m_{\Lambda_4^0} = 1890$  MeV and  $m_{\Sigma_4^0} = 1840$  MeV the numerical value of the contribution of the resonances  $\Lambda_4^0$  and  $\Sigma_4^0$  to the P-wave scattering length  $a_{3/2}^{K^-p}$  reads

$$\sum_{R=\Lambda_4^0, \Sigma_4^0} (a_{3/2}^{K^-p})_R = -0.001 \text{ m}_\pi^{-3}. \tag{3.13}$$

The P-wave scattering length of  $K^-p$  scattering with total angular momentum  $J = 3/2$  is given by

$$a_{3/2}^{K^-p} = (a_{3/2}^{K^-p})_{SKT} + 0.059 \text{ m}_\pi^{-3}. \tag{3.14}$$

Summing up the contributions (3.8) and (3.14) we obtain the total P-wave scattering length of elastic  $K^-p$  scattering in the P-wave state

$$2 a_{3/2}^{K^-p} + a_{1/2}^{K^-p} = (2 a_{3/2}^{K^-p} + a_{1/2}^{K^-p})_{SKT} + 0.111 \text{ m}_\pi^{-3}. \tag{3.15}$$

Now we turn to the calculation of the term  $(2 a_{3/2}^{K^-p} + a_{1/2}^{K^-p})_{SKT}$ .

### 3.2 Soft-kaon theorem for amplitude of elastic $K^-p$ scattering and elastic P-wave background

Soft-kaon theorems, as a part of ChPT [4, 5], define amplitudes of low-energy reactions with kaons as expansions in powers of 4-momenta of kaons  $k$ , with kaons treated off-mass shell  $k^2 \neq m_K^2$ . Using the reduction technique and the PCAC hypothesis [23]–[30] the  $S$ -matrix element of elastic low-energy transition  $K^-p \rightarrow K^-p$  can be defined by

$$\begin{aligned}
& \langle out; K^-(\vec{k})p(-\vec{k}, \sigma_p) | K^-(\vec{q})p(-\vec{q}, \sigma_p); in \rangle = -\frac{(m_K^2 - k^2)}{\sqrt{2} F_K m_K^2} \frac{(m_K^2 - q^2)}{\sqrt{2} F_K m_K^2} \\
& \times \int d^4x d^4y e^{+ik \cdot x - iq \cdot y} \langle p(-\vec{k}, \sigma_p) | T(\partial^\mu J_{5\mu}^{4+i5}(x) \partial^\nu J_{5\nu}^{4-i5}(y)) | p(-\vec{q}, \sigma_p) \rangle, \tag{3.16}
\end{aligned}$$

where  $T$  is a time-ordering operator and  $J_{5\mu}^{4+i5}(x)$  and  $J_{5\nu}^{4-i5}(x)$  are axial-vector hadronic currents with quantum numbers of the  $K^-$  and  $K^+$  mesons [23, 25];  $F_K = 113$  MeV is the

PCAC constant of charged  $K$  mesons. For further reduction of the r.h.s. of Eq.(3.16) we use the relation [23]

$$\begin{aligned} T(\partial^\mu J_{5\mu}^{4+i5}(x) \partial^\nu J_{5\nu}^{4-i5}(y)) &= \frac{\partial}{\partial x_\mu} \frac{\partial}{\partial y_\nu} T(J_{5\mu}^{4+i5}(x) J_{5\nu}^{4-i5}(y)) \\ &- \frac{1}{2} \frac{\partial}{\partial x_\mu} \left\{ \delta(x^0 - y^0) [J_{50}^{4-i5}(y), J_{5\mu}^{4+i5}(x)] \right\} - \frac{1}{2} \frac{\partial}{\partial y_\nu} \left\{ \delta(x^0 - y^0) [J_{50}^{4+i5}(x), J_{5\nu}^{4-i5}(y)] \right\} \\ &- \frac{1}{2} \delta(x^0 - y^0) [J_{50}^{4-i5}(y), \partial^\mu J_{5\mu}^{4+i5}(x)] - \frac{1}{2} \delta(x^0 - y^0) [J_{50}^{4+i5}(x), \partial^\nu J_{5\nu}^{4-i5}(y)]. \end{aligned} \quad (3.17)$$

Substituting (3.17) into (3.16) and making integration by parts and dropping surface terms we arrive at the expression

$$\begin{aligned} \langle out; K^-(\vec{k}) p(-\vec{k}, \sigma_p) | K^-(\vec{q}) p(-\vec{q}, \sigma_p); in \rangle &= - \frac{(m_K^2 - k^2)}{\sqrt{2} F_K m_K^2} \frac{(m_K^2 - q^2)}{\sqrt{2} F_K m_K^2} \\ &\times \int d^4x d^4y e^{+ik \cdot x - iq \cdot y} \left\{ k^\mu q^\nu \langle p(-\vec{k}, \sigma_p) | T(J_{5\mu}^{4+i5}(x) J_{5\nu}^{4-i5}(y)) | p(-\vec{q}, \sigma_p) \rangle \right. \\ &+ \frac{1}{2} i k^\mu \delta(x^0 - y^0) \langle p(-\vec{k}, \sigma_p) | [J_{50}^{4-i5}(y), J_{5\mu}^{4+i5}(x)] | p(-\vec{q}, \sigma_p) \rangle \\ &- \frac{1}{2} i q^\nu \delta(x^0 - y^0) \langle p(-\vec{k}, \sigma_p) | [J_{50}^{4+i5}(x), J_{5\nu}^{4-i5}(y)] | p(-\vec{q}, \sigma_p) \rangle \\ &- \frac{1}{2} \delta(x^0 - y^0) \langle p(-\vec{k}, \sigma_p) | [J_{50}^{4-i5}(y), \partial^\mu J_{5\mu}^{4+i5}(x)] | p(-\vec{q}, \sigma_p) \rangle \\ &\left. - \frac{1}{2} \delta(x^0 - y^0) \langle p(-\vec{k}, \sigma_p) | [J_{50}^{4+i5}(x), \partial^\nu J_{5\nu}^{4-i5}(y)] | p(-\vec{q}, \sigma_p) \rangle \right\} \end{aligned} \quad (3.18)$$

From (3.18) we obtain the amplitude of elastic low-energy  $K^- p$  scattering with  $K^-$  mesons off-mass shell. It reads

$$\begin{aligned} M(K^-(\vec{q}) p(-\vec{q}, \sigma_p) \rightarrow K^-(\vec{k}) p(-\vec{k}, \sigma_p)) &= \frac{(m_K^2 - k^2)}{\sqrt{2} F_K m_K^2} \frac{(m_K^2 - q^2)}{\sqrt{2} F_K m_K^2} \\ &\times i \int d^4x e^{+ik \cdot x} \left\{ k^\mu q^\nu \langle p(-\vec{k}, \sigma_p) | T(J_{5\mu}^{4+i5}(x) J_{5\nu}^{4-i5}(0)) | p(-\vec{q}, \sigma_p) \rangle \right. \\ &+ \frac{1}{2} i k^\mu \delta(x^0) \langle p(-\vec{k}, \sigma_p) | [J_{50}^{4-i5}(0), J_{5\mu}^{4+i5}(x)] | p(-\vec{q}, \sigma_p) \rangle \\ &- \frac{1}{2} i q^\nu \delta(x^0) \langle p(-\vec{k}, \sigma_p) | [J_{50}^{4+i5}(x), J_{5\nu}^{4-i5}(0)] | p(-\vec{q}, \sigma_p) \rangle \\ &- \frac{1}{2} \delta(x^0) \langle p(-\vec{k}, \sigma_p) | [J_{50}^{4-i5}(0), \partial^\mu J_{5\mu}^{4+i5}(x)] | p(-\vec{q}, \sigma_p) \rangle \\ &\left. - \frac{1}{2} \delta(x^0) \langle p(-\vec{k}, \sigma_p) | [J_{50}^{4+i5}(x), \partial^\nu J_{5\nu}^{4-i5}(0)] | p(-\vec{q}, \sigma_p) \rangle \right\}. \end{aligned} \quad (3.19)$$

The equal-time commutators read [23, 25]

$$\begin{aligned} \delta(x^0) [J_{50}^{4+i5}(x), J_{5\nu}^{4-i5}(0)] &= (J_\nu^3(0) + \sqrt{3} J_\nu^8(0)) \delta^{(4)}(x), \\ \delta(x^0) [J_{50}^{4+i5}(x), \partial^\nu J_{5\nu}^{4-i5}(0)] &= -i (\sigma_{44}(0) + \sigma_{55}(0)) \delta^{(4)}(x), \end{aligned} \quad (3.20)$$

where  $J_\nu^3(0)$  and  $J_\nu^8(0)$  are vector hadronic currents, related to the electromagnetic  $J_\nu^{(em)}(0)$  and hypercharge  $Y_\nu(0)$  currents by

$$J_\nu^3(0) + \sqrt{3} J_\nu^8(0) = J_{nu}^{(em)}(0) + Y_\nu(0), \quad (3.21)$$

and  $\sigma_{ab}(0)$  is so-called  $\sigma$ -term operator. The  $\sigma$ -term operator  $\sigma_{ab}(0)$  is related to the breaking of chiral symmetry. It can also be defined by the double commutator [26]:  $\sigma_{ab}(0) = [Q_5^a(0), [Q_5^b(0), H_{\chi SB}(0)]]$ , where  $Q_5^a(0)$  is the axial-vector charge operator and  $H_{\chi SB}$  is the Hamiltonian of strong interactions breaking of chiral symmetry. In terms of current quark fields it reads  $H_{\chi SB}(0) = m_u \bar{u}(0)u(0) + m_d \bar{d}(0)d(0) + m_s \bar{s}(0)s(0)$ , where  $m_q$  ( $q = u, d, s$ ) and  $q(0) = u(0), d(0), s(0)$  are masses and interpolating fields of current quarks.

Substituting (3.20) into (3.19) and using (3.21) we get

$$\begin{aligned} M(K^-(\vec{q})p(-\vec{q}, \sigma_p) \rightarrow K^-(\vec{k})p(-\vec{k}, \sigma_p)) &= \frac{(m_K^2 - k^2)}{\sqrt{2} F_K m_K^2} \frac{(m_K^2 - q^2)}{\sqrt{2} F_K m_K^2} \\ &\times \left\{ k^\mu q^\nu i \int d^4x e^{+ik \cdot x} \langle p(-\vec{k}, \sigma_p) | T(J_{5\mu}^{4+i5}(x) J_{5\nu}^{4-i5}(0)) | p(-\vec{q}, \sigma_p) \rangle \right. \\ &+ \frac{1}{2} (k^\mu + q^\mu) \langle p(-\vec{k}, \sigma_p) | J_\mu^{(\text{em})}(0) + Y_\mu(0) | p(-\vec{q}, \sigma_p) \rangle \\ &\left. - \langle p(-\vec{k}, \sigma_p) | \sigma_{44}(0) + \sigma_{55}(0) | p(-\vec{q}, \sigma_p) \rangle \right\}. \end{aligned} \quad (3.22)$$

The matrix elements of the  $\sigma$ -term operator can be represented by [27]–[30, 38]

$$\langle p(-\vec{k}, \sigma_p) | \sigma_{44}(0) + \sigma_{55}(0) | p(-\vec{q}, \sigma_p) \rangle = 2 \sigma_{KN}^{(I=1)}(t) \bar{u}(-\vec{k}, \sigma_p) u(-\vec{q}, \sigma_p), \quad (3.23)$$

where  $\sigma_{KN}^{(I=1)}(t)$  is the scalar form factor [26]–[30, 38], defining the contribution to the amplitude of  $\bar{K}N$  scattering in the state with isospin  $I = 1$ , and  $t = -(\vec{k} - \vec{q})^2$  is a squared transferred momentum. In terms of the quark-field operators the  $\sigma$ -term  $\sigma_{KN}^{(I=1)}(t)$  is defined by [26, 27, 30, 38]

$$\sigma_{KN}^{(I=1)}(t) = \frac{m_u + m_s}{4m_N} \langle p(-\vec{k}, \sigma_p) | \bar{u}(0)u(0) + \bar{s}(0)s(0) | p(-\vec{q}, \sigma_p) \rangle. \quad (3.24)$$

According to ChPT [4, 5], the  $\sigma$ -term is of order of squared 4-momenta of  $K^-$ -mesons, i.e.  $\sigma_{KN}^{(I=1)}(t) \sim k^2 \sim q^2$ .

Accounting for the contribution of the  $K^-$ -meson pole and keeping the terms of order of  $O(k^2)$  and  $O(q^2)$  inclusively, we get the following expression for the amplitude of elastic low-energy  $K^-p$  scattering [30]

$$\begin{aligned} M(K^-(\vec{q})p(-\vec{q}, \sigma_p) \rightarrow K^-(\vec{k})p(-\vec{k}, \sigma_p)) &= \\ &= \bar{u}(-\vec{k}, \sigma_p) \left\{ \frac{F_E^p(t) + F_Y^p(t)}{4F_K^2} (k + q)^\mu \gamma_\mu - \frac{1}{F_K^2} \left[ \sigma_{KN}^{(I=1)}(t) - k^\mu q^\nu W_{\mu\nu}(\vec{k}, \vec{q}) \right] \right\} u(-\vec{q}, \sigma_p), \end{aligned} \quad (3.25)$$

where we have denoted

$$\begin{aligned} \langle p(-\vec{k}, \sigma_p) | J_\mu^{(\text{em})}(0) + Y_\mu(0) | p(-\vec{q}, \sigma_p) \rangle &= (F_E^p(t) + F_Y^p(t)) \bar{u}(-\vec{k}, \sigma_p) \gamma_\mu u(-\vec{q}, \sigma_p), \\ \frac{1}{2} i \int d^4x \langle p(-\vec{k}, \sigma_p) | T(J_{5\mu}^{4+i5}(x) J_{5\nu}^{4-i5}(0)) | p(-\vec{q}, \sigma_p) \rangle &= \\ &= \bar{u}(-\vec{k}, \sigma_p) W_{\mu\nu}(\vec{k}, \vec{q}) u(-\vec{q}, \sigma_p). \end{aligned} \quad (3.26)$$

Here  $F_E^p(t)$  and  $F_Y^p(t)$  are the form factors of the electric and hypercharge of the proton, normalized by  $F_E^p(0) = F_Y^p(0) = 1$ . We have not taken into account the magnetic form factor, which does not contribute to the S- and P-wave amplitudes of  $K^-p$  scattering at threshold.

The last two terms in Eq.(3.25) are of order of  $O(k^2)$ , where  $k^2 \sim q^2 \sim k \cdot q$ . For the calculation of the P-wave scattering length of elastic  $K^-p$  scattering the contribution of the terms of order of  $O(k^2)$  can be neglected.

From Eq.(3.25) at leading order in chiral expansion [4, 5] we obtain the contribution to the P-wave amplitude of low-energy elastic  $K^-p$  scattering

$$(2a_{3/2}^{K^-p} + a_{1/2}^{K^-p})_{SKT} = \frac{1}{16\pi} \frac{\mu}{F_K^2} \frac{1}{m_N^2} = 0.002 \text{ m}_\pi^{-3}. \quad (3.27)$$

Hence, the P-wave scattering length  $(2a_{3/2}^{K^-p} + a_{1/2}^{K^-p})_{SKT}$  is smaller than the contribution of the resonance states and practically can be neglected for the calculation of the P-wave scattering lengths of elastic  $K^-p$  scattering and, correspondingly, for the calculation of the energy level shift of the  $np$  excited state of kaonic hydrogen. This implies that the P-wave scattering lengths  $(2a_{3/2}^{Y\pi} + a_{1/2}^{Y\pi})_{SKT}$  can also be neglected in comparison with the contributions of the resonance states.

### 3.3 P-wave scattering length $2a_{3/2}^{K^-p} + a_{1/2}^{K^-p}$ of elastic $K^-p$ scattering and energy level shift of $np$ excited state of kaonic hydrogen

Substituting (3.27) into (3.15) we obtain the P-wave scattering length of elastic  $K^-p$  scattering

$$2a_{3/2}^{K^-p} + a_{1/2}^{K^-p} = 0.113 \text{ m}_\pi^{-3}. \quad (3.28)$$

Using (3.28) we compute the shift of the energy level of the  $np$  excited state of kaonic hydrogen, given by Eq.(2.13). We get

$$\epsilon_{np} = \frac{32}{3} \frac{1}{n^3} \left(1 - \frac{1}{n^2}\right) \epsilon_{2p}, \quad (3.29)$$

where the shift of the energy level of the  $2p$  excited state is equal to

$$\epsilon_{np} = -\frac{\alpha^5}{16} \left(\frac{m_K m_N}{m_K + m_N}\right)^4 (2a_{3/2}^{K^-p} + a_{1/2}^{K^-p}) = -0.6 \text{ meV}. \quad (3.30)$$

Hence, the shift of the energy level  $\epsilon_{np}$  of the  $np$  excited state of kaonic hydrogen, induced by strong low-energy interactions, is smaller than 1 meV, i.e.  $|\epsilon_{np}| < 1 \text{ meV}$ .

We would like to emphasize that unlike the shift of the energy level of the  $ns$  state of kaonic hydrogen, which is defined by repulsive forces  $\epsilon_{ns} = (203 \pm 15)/n^3 \text{ eV}$  [1], the shift of the energy level of the  $np$  excited state  $\epsilon_{np}$ , given by Eq.(3.29), is caused by attractive forces.

## 4 P-wave scattering lengths $2a_{3/2}^{Y\pi} + a_{1/2}^{Y\pi}$ of inelastic channels $K^-p \rightarrow Y\pi$

The imaginary part of the P-wave amplitude of elastic  $K^-p$  scattering at threshold, defining the total width of the excited  $np$  state of kaonic hydrogen, is caused by the four opened inelastic channels  $K^-p \rightarrow \Sigma^+\pi^-$ ,  $K^-p \rightarrow \Sigma^-\pi^+$ ,  $K^-p \rightarrow \Sigma^0\pi^0$  and  $K^-p \rightarrow \Lambda^0\pi^0$ . At threshold the contribution of these inelastic channels we describe by the P-wave scattering lengths  $a_{1/2}^{Y\pi}$  and  $a_{3/2}^{Y\pi}$  with  $Y\pi = \Sigma^+\pi^-$ ,  $\Sigma^-\pi^+$ ,  $\Sigma^0\pi^0$  and  $\Lambda^0\pi^0$ , respectively. The P-wave scattering lengths  $a_{1/2}^{Y\pi}$  and  $a_{3/2}^{Y\pi}$  determine low-energy transitions  $K^-p \rightarrow Y\pi$  with total angular momentum  $J = 1/2$  and  $J = 3/2$ , respectively.

The P-wave scattering lengths  $a_J^{Y\pi}$  we represent in the form of the superposition of the background part  $(a_J^{Y\pi})_B$  and the resonant part  $\sum_R (a_J^{Y\pi})_R$ . It is convenient to include the contribution of the octet of low-lying baryons  $B(\mathbf{8}) = (N(940), \Lambda^0(1116), \Sigma(1193))$  to the resonant part and to define the contribution of the background as  $(a_J^{Y\pi})_B = (a_J^{Y\pi})_{SKT}$ . Since, as has been shown above, the contribution of the resonances  $\Lambda^0(1890)$  and  $\Sigma^0(1840)$  is negligible small relative to the contribution of the resonance  $\Sigma^0(1385)$ , below for the calculation of the P-wave scattering lengths of inelastic channels  $K^-p \rightarrow Y\pi$  we do not take them into account.

### 4.1 P-wave scattering lengths $2a_{3/2}^{\Sigma^+\pi^-} + a_{1/2}^{\Sigma^+\pi^-}$ of inelastic channel $K^-p \rightarrow \Sigma^+\pi^-$

The resonant parts of the P-wave scattering lengths  $a_{1/2}^{\Sigma^+\pi^-}$  and  $a_{3/2}^{\Sigma^+\pi^-}$  of the reaction  $K^-p \rightarrow \Sigma^+\pi^-$  are equal to

$$\begin{aligned}
\sum_R (a_{1/2}^{\Sigma^+\pi^-})_R &= \\
&= \frac{1}{8\pi} \frac{1}{m_K + m_N} \left[ \frac{1}{\sqrt{3}} (3 - 2\alpha) g_{\pi NN} \right] \left[ \frac{2}{\sqrt{3}} \alpha g_{\pi NN} \right] \frac{1}{2\sqrt{m_\Sigma m_N}} \frac{1}{m_{\Lambda^0} - m_K - m_N} \\
&+ \frac{1}{8\pi} \frac{1}{m_K + m_N} \left[ \frac{1}{\sqrt{3}} (3 - 2\alpha_1) g_{\pi NN_1} \right] \left[ \frac{2}{\sqrt{3}} \alpha_1 g_{\pi NN_1} \right] \frac{1}{2\sqrt{m_\Sigma m_N}} \frac{1}{m_{\Lambda_1^0} - m_K - m_N} \\
&+ \frac{1}{8\pi} \frac{1}{m_K + m_N} \left[ \frac{1}{\sqrt{3}} (3 - 2\alpha_2) g_{\pi NN_2} \right] \left[ \frac{2}{\sqrt{3}} \alpha_2 g_{\pi NN_2} \right] \frac{1}{2\sqrt{m_\Sigma m_N}} \frac{1}{m_{\Lambda_2^0} - m_K - m_N} \\
&+ \frac{1}{8\pi} \frac{1}{m_K + m_N} \left[ (2\alpha - 1) g_{\pi NN} \right] \left[ 2(1 - \alpha) g_{\pi NN} \right] \frac{1}{2\sqrt{m_\Sigma m_N}} \frac{1}{m_{\Sigma^0} - m_K - m_N} \\
&+ \frac{1}{8\pi} \frac{1}{m_K + m_N} \left[ (2\alpha_1 - 1) g_{\pi NN_1} \right] \left[ 2(1 - \alpha_1) g_{\pi NN_1} \right] \frac{1}{2\sqrt{m_\Sigma m_N}} \frac{1}{m_{\Sigma_1^0} - m_K - m_N} \\
&+ \frac{1}{8\pi} \frac{1}{m_K + m_N} \left[ (2\alpha_2 - 1) g_{\pi NN_2} \right] \left[ 2(1 - \alpha_2) g_{\pi NN_2} \right] \frac{1}{2\sqrt{m_\Sigma m_N}} \frac{1}{m_{\Sigma_2^0} - m_K - m_N} \\
&= (-0.015 + 0.006 - 0.001 - 0.005 + 0.001 - 0.002) m_\pi^{-3} = -0.016 m_\pi^{-3}. \quad (4.1)
\end{aligned}$$

and

$$(a_{3/2}^{\Sigma^+\pi^-})_R = \frac{g_{\pi NN}^2}{36\pi m_N} \frac{1}{m_K + m_N} \frac{1}{m_{\Sigma_3^0} - m_N - m_K} \sqrt{\frac{m_\Sigma}{m_N}} \left( 1 + \frac{1}{4} \frac{m_K}{m_N} \frac{m_K + m_N}{m_{\Sigma_3^0}} \right) =$$

$$= -0.082 \text{ m}_\pi^{-3}. \quad (4.2)$$

The total P-wave scattering length of the reaction  $K^-p \rightarrow \Sigma^+\pi^-$  is equal to

$$2a_{3/2}^{\Sigma^+\pi^-} + a_{1/2}^{\Sigma^+\pi^-} = (2a_{3/2}^{\Sigma^+\pi^-} + a_{1/2}^{\Sigma^+\pi^-})_{SKT} - 0.180 \text{ m}_\pi^{-3}. \quad (4.3)$$

## 4.2 P-wave scattering lengths of $2a_{3/2}^{\Sigma^-\pi^+} + a_{1/2}^{\Sigma^-\pi^+}$ of inelastic channel $K^-p \rightarrow \Sigma^-\pi^+$

The resonant parts of the P-wave scattering lengths  $a_{1/2}^{\Sigma^-\pi^+}$  and  $a_{3/2}^{\Sigma^-\pi^+}$  of the reaction  $K^-p \rightarrow \Sigma^-\pi^+$  are equal to

$$\begin{aligned} \sum_R (a_{1/2}^{\Sigma^-\pi^+})_R &= \\ &= \frac{1}{8\pi} \frac{1}{m_K + m_N} \left[ \frac{1}{\sqrt{3}} (3 - 2\alpha) g_{\pi NN} \right] \left[ \frac{2}{\sqrt{3}} \alpha g_{\pi NN} \right] \frac{1}{2\sqrt{m_\Sigma m_N}} \frac{1}{m_{\Lambda^0} - m_K - m_N} \\ &+ \frac{1}{8\pi} \frac{1}{m_K + m_N} \left[ \frac{1}{\sqrt{3}} (3 - 2\alpha_1) g_{\pi NN_1} \right] \left[ \frac{2}{\sqrt{3}} \alpha_1 g_{\pi NN_1} \right] \frac{1}{2\sqrt{m_\Sigma m_N}} \frac{1}{m_{\Lambda_1^0} - m_K - m_N} \\ &+ \frac{1}{8\pi} \frac{1}{m_K + m_N} \left[ \frac{1}{\sqrt{3}} (3 - 2\alpha_2) g_{\pi NN_2} \right] \left[ \frac{2}{\sqrt{3}} \alpha_2 g_{\pi NN_2} \right] \frac{1}{2\sqrt{m_\Sigma m_N}} \frac{1}{m_{\Lambda_2^0} - m_K - m_N} \\ &- \frac{1}{8\pi} \frac{1}{m_K + m_N} \left[ (2\alpha - 1) g_{\pi NN} \right] \left[ 2(1 - \alpha) g_{\pi NN} \right] \frac{1}{2\sqrt{m_\Sigma m_N}} \frac{1}{m_{\Sigma^0} - m_K - m_N} \\ &- \frac{1}{8\pi} \frac{1}{m_K + m_N} \left[ (2\alpha_1 - 1) g_{\pi NN_1} \right] \left[ 2(1 - \alpha_1) g_{\pi NN_1} \right] \frac{1}{2\sqrt{m_\Sigma m_N}} \frac{1}{m_{\Sigma_1^0} - m_K - m_N} \\ &- \frac{1}{8\pi} \frac{1}{m_K + m_N} \left[ (2\alpha_2 - 1) g_{\pi NN_2} \right] \left[ 2(1 - \alpha_2) g_{\pi NN_2} \right] \frac{1}{2\sqrt{m_\Sigma m_N}} \frac{1}{m_{\Sigma_2^0} - m_K - m_N} \\ &= (-0.015 + 0.006 - 0.001 + 0.005 - 0.001 + 0.002) \text{ m}_\pi^{-3} = -0.004 \text{ m}_\pi^{-3}. \end{aligned} \quad (4.4)$$

and

$$(a_{3/2}^{\Sigma^-\pi^+})_R = -(a_{3/2}^{\Sigma^+\pi^-})_R = +0.082 \text{ m}_\pi^{-3}. \quad (4.5)$$

The total P-wave scattering length of the reaction  $K^-p \rightarrow \Sigma^-\pi^+$  is equal to

$$2a_{3/2}^{\Sigma^-\pi^+} + a_{1/2}^{\Sigma^-\pi^+} = (2a_{3/2}^{\Sigma^-\pi^+} + a_{1/2}^{\Sigma^-\pi^+})_{SKT} + 0.160 \text{ m}_\pi^{-3}. \quad (4.6)$$

## 4.3 P-wave scattering lengths of $2a_{3/2}^{\Sigma^0\pi^0} + a_{1/2}^{\Sigma^0\pi^0}$ of inelastic channel $K^-p \rightarrow \Sigma^0\pi^0$

The resonant parts of the P-wave scattering lengths  $a_{1/2}^{\Sigma^0\pi^0}$  and  $a_{3/2}^{\Sigma^0\pi^0}$  of the reaction  $K^-p \rightarrow \Sigma^0\pi^0$  are equal to

$$\begin{aligned} \sum_R (a_{1/2}^{\Sigma^0\pi^0})_R &= \\ &= \frac{1}{8\pi} \frac{1}{m_K + m_N} \left[ \frac{1}{\sqrt{3}} (3 - 2\alpha) g_{\pi NN} \right] \left[ \frac{2}{\sqrt{3}} \alpha g_{\pi NN} \right] \frac{1}{2\sqrt{m_\Sigma m_N}} \frac{1}{m_{\Lambda^0} - m_K - m_N} \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{8\pi} \frac{1}{m_K + m_N} \left[ \frac{1}{\sqrt{3}} (3 - 2\alpha_1) g_{\pi NN_1} \right] \left[ \frac{2}{\sqrt{3}} \alpha_1 g_{\pi NN_1} \right] \frac{1}{2\sqrt{m_\Sigma m_N}} \frac{1}{m_{\Lambda_1^0} - m_K - m_N} \\
& + \frac{1}{8\pi} \frac{1}{m_K + m_N} \left[ \frac{1}{\sqrt{3}} (3 - 2\alpha_2) g_{\pi NN_2} \right] \left[ \frac{2}{\sqrt{3}} \alpha_2 g_{\pi NN_2} \right] \frac{1}{2\sqrt{m_\Sigma m_N}} \frac{1}{m_{\Lambda_2^0} - m_K - m_N} \\
& = (-0.015 + 0.006 - 0.001) m_\pi^{-3} = -0.010 m_\pi^{-3}.
\end{aligned} \tag{4.7}$$

and

$$(a_{3/2}^{\Sigma^0 \pi^0})_R = 0. \tag{4.8}$$

The total P-wave scattering length of the reaction  $K^- p \rightarrow \Sigma^0 \pi^0$  is equal to

$$2 a_{3/2}^{\Sigma^0 \pi^0} + a_{1/2}^{\Sigma^0 \pi^0} = (2 a_{3/2}^{\Sigma^0 \pi^0} + a_{1/2}^{\Sigma^0 \pi^0})_{SKT} - 0.010 m_\pi^{-3}. \tag{4.9}$$

#### 4.4 P-wave scattering lengths of $2a_{3/2}^{\Lambda^0 \pi^0} + a_{1/2}^{\Lambda^0 \pi^0}$ of inelastic channel $K^- p \rightarrow \Lambda^0 \pi^0$

The resonant parts of the P-wave scattering lengths  $a_{1/2}^{\Lambda^0 \pi^0}$  and  $a_{3/2}^{\Lambda^0 \pi^0}$  of the reaction  $K^- p \rightarrow \Lambda^0 \pi^0$  are equal to

$$\begin{aligned}
& \sum_R (a_{1/2}^{\Lambda^0 \pi^0})_R = \\
& = \frac{1}{8\pi} \frac{1}{m_K + m_N} \left[ - (2\alpha - 1) g_{\pi NN} \right] \left[ \frac{2}{\sqrt{3}} \alpha g_{\pi NN} \right] \frac{1}{2\sqrt{m_{\Lambda^0} m_N}} \frac{1}{m_{\Sigma^0} - m_K - m_N} \\
& + \frac{1}{8\pi} \frac{1}{m_K + m_N} \left[ - (2\alpha_1 - 1) g_{\pi NN_1} \right] \left[ \frac{2}{\sqrt{3}} \alpha_1 g_{\pi NN_1} \right] \frac{1}{2\sqrt{m_{\Lambda^0} m_N}} \frac{1}{m_{\Sigma_1^0} - m_K - m_N} \\
& + \frac{1}{8\pi} \frac{1}{m_K + m_N} \left[ - (2\alpha_2 - 1) g_{\pi NN_2} \right] \left[ \frac{2}{\sqrt{3}} \alpha_2 g_{\pi NN_2} \right] \frac{1}{2\sqrt{m_{\Lambda^0} m_N}} \frac{1}{m_{\Sigma_2^0} - m_K - m_N} \\
& = (0.006 - 0.005 - 0.001) m_\pi^{-3} = 0.
\end{aligned} \tag{4.10}$$

and

$$\begin{aligned}
(a_{3/2}^{\Lambda^0 \pi^0})_R & = \frac{\sqrt{3} g_{\pi NN}^2}{36\pi m_N} \frac{1}{m_K + m_N} \frac{1}{m_{\Sigma_3^0} - m_N - m_K} \sqrt{\frac{m_{\Lambda^0}}{m_N}} \left( 1 + \frac{1}{4} \frac{m_K}{m_N} \frac{m_K + m_N}{m_{\Sigma_3^0}} \right) = \\
& = -0.137 m_\pi^{-3}.
\end{aligned} \tag{4.11}$$

The total P-wave scattering length of the reaction  $K^- p \rightarrow \Lambda^0 \pi^0$  is equal to

$$2 a_{3/2}^{\Lambda^0 \pi^0} + a_{1/2}^{\Lambda^0 \pi^0} = (2 a_{3/2}^{\Lambda^0 \pi^0} + a_{1/2}^{\Lambda^0 \pi^0})_{SKT} - 0.274 m_\pi^{-3}. \tag{4.12}$$

#### 4.5 P-wave scattering lengths of inelastic reactions $K^- p \rightarrow Y \pi$ and energy level width of $np$ excited state of kaonic hydrogen

According to the estimate Eq.(3.27), the contribution of the P-wave scattering lengths  $(2 a_{3/2}^{Y \pi} + a_{1/2}^{Y \pi})_{SKT}$  can be neglected in comparison with the contribution of the baryon

resonances. Therefore, below we neglect  $(2a_{3/2}^{Y\pi} + a_{1/2}^{Y\pi})_{SKT}$  for the estimate the energy level width of the  $np$  excited state of kaonic hydrogen.

Using Eqs.(4.3), (4.6) and (4.12) and substituting them into Eq.(2.13), we compute the energy level width of the  $np$  excited state of kaonic hydrogen

$$\Gamma_{np} = \frac{32}{3} \frac{1}{n^3} \left(1 - \frac{1}{n^2}\right) \Gamma_{2p}, \quad (4.13)$$

The partial width  $\Gamma_{2p}$  of the energy level of the  $2p$  excited state of kaonic hydrogen is equal to

$$\Gamma_{2p} = \frac{\alpha^5}{24} \left(\frac{m_K m_N}{m_K + m_N}\right)^4 \sum_{Y\pi} (2a_{3/2}^{Y\pi} + a_{1/2}^{Y\pi})^2 k_{Y\pi}^3 = 2 \text{ meV} \quad (4.14)$$

or  $\Gamma_{2p} = 3 \times 10^{12} \text{ sec}^{-1}$ .

The lifetime of the  $2p$  state of kaonic hydrogen, defined by the decays of kaonic hydrogen into hadronic states  $(K^-p)_{2p} \rightarrow Y\pi$ , where  $Y\pi = \Sigma^+\pi^-, \Sigma^-\pi^+, \Sigma^0\pi^0$  and  $\Lambda^0\pi^0$ , is equal to  $\tau_{2p} = 3.4 \times 10^{-13} \text{ sec}$ . It is much smaller than the lifetime of the  $K^-$ -meson,  $\tau_{K^-} = 1.24 \times 10^{-8} \text{ sec}$  [20], which is the upper limit on the lifetime of kaonic hydrogen. Thus, the rates of the hadronic decays of kaonic hydrogen in the  $np$  excited states are comparable with the rates of the de-excitation of kaonic hydrogen  $np \rightarrow 1s$ , caused by the emission of the  $X$ -rays [7]–[13].

The result obtained for the partial width of the excited  $2p$  state of kaonic hydrogen, given by Eq.(4.14), is important for the theoretical analysis of the  $X$ -ray yields in kaonic hydrogen [7]–[13].

## 5 Conclusion

The quantum field theoretic model of the description of low-energy  $\bar{K}N$  interaction in the S-wave state near threshold, which we have suggested in [1, 6], is extended on the analysis of low-energy  $\bar{K}N$  interactions in the P-wave state near threshold. We would like to emphasize that our approach to the description of low-energy  $\bar{K}N$  interaction in the S-wave state near threshold agrees well with the non-relativistic Effective Field Theory based on ChPT by Gasser and Leutwyler, which has been recently applied by Meißner *et al.* [3] to the calculation of the energy level displacement of the  $ns$  state of kaonic hydrogen and systematic corrections to the energy level displacement of the  $ns$  state, caused by QCD isospin-breaking and electromagnetic interactions. The result for the energy level displacement of the  $ns$  state of kaonic hydrogen has been obtained in [3] in terms of the S-wave scattering lengths  $a_0^0$  and  $a_0^1$  of  $\bar{K}N$  scattering with isospin  $I = 0$  and  $I = 1$ , respectively. The S-wave scattering lengths  $a_0^0$  and  $a_0^1$  have been treated as free parameters of the approach. Using our results for the S-wave scattering lengths  $a_0^0$  and  $a_0^1$  [1, 6] and keeping leading terms in QCD isospin-breaking and electromagnetic interactions, i.e. accounting for only the contribution of Coulombic photons, we have shown that the numerical value of the energy level displacement of the  $ns$  state of kaonic hydrogen, computed by Meißner *et al.* [3], agrees well with both our theoretical prediction [1] and recent experimental data by the DEAR Collaboration [2] within 1.5 standard deviations for the shift and one standard deviation for the width. Hence, our approach to



the description low-energy dynamics of strong low-energy  $\bar{K}N$  interactions at threshold agrees well with general description of strong low-energy interactions of hadrons within non-relativistic Effective Field Theory based on ChPT [3]–[5].

The detection of the  $X$ -rays of the  $X$ -ray cascade processes, leading to the de-excitation of kaonic hydrogen from the excited states to the ground state, is the main experimental tool for the measurement of the energy level displacement of the ground state of kaonic hydrogen, caused by strong low-energy interactions [2, 39]. The main transitions in kaonic hydrogen, which are measured experimentally for the extraction of the energy level displacement of the ground state, are  $3p \rightarrow 1s$  and  $2p \rightarrow 1s$ , i.e. the reactions  $(K^-p)_{3p} \rightarrow (K^-p)_{1s} + \gamma$  and  $(K^-p)_{2p} \rightarrow (K^-p)_{1s} + \gamma$ .

As has been pointed out by Markushin and Jensen the yields of  $X$ -rays of these transitions are quite sensitive to the value of  $\Gamma_{2p}$  [13]. Using  $\Gamma_{2p}$  as an input parameter taking values from the region  $0.1 \text{ meV} \leq \Gamma_{2p} \leq 0.9 \text{ meV}$ , Markushin and Jensen [13] have found that their theoretical predictions for the  $X$ -ray yields in kaonic hydrogen agree well with the experimental data on the  $X$ -ray yields detected by the KEK Collaboration [40], which have been used for the extraction of the energy level displacement of the ground state of kaonic hydrogen, for  $\Gamma_{2p} = 0.3 \text{ meV} = 4.6 \times 10^{11} \text{ sec}^{-1}$  and  $\epsilon_{1s} = 320 \text{ eV}$  and  $\Gamma_{1s} = 400 \text{ eV}$ .

Recent experimental data on the energy level displacement of the ground of kaonic hydrogen obtained by the DEAR Collaboration [2] by a factor of 2 smaller than the experimental data by the KEK Collaboration [40]. Our theoretical analysis of the energy level displacement of the  $2p$  excited state of kaonic hydrogen has shown that the rate of the hadronic decays of kaonic hydrogen from the  $2p$  excited state is equal to  $\Gamma_{2p} = 2 \text{ meV} = 3 \times 10^{12} \text{ sec}^{-1}$ , which is an order of magnitude larger than the phenomenological value  $\Gamma_{2p} = 0.3 \text{ meV} = 4.6 \times 10^{11} \text{ sec}^{-1}$ , used by Markushin and Jensen as an input parameter [13].

Thus, the computed value  $\Gamma_{2p} = 2 \text{ meV}$  of the energy level of the  $2p$  excited state of kaonic hydrogen can be applied to the theoretical analysis of the  $X$ -ray yields in kaonic hydrogen of recent experimental data by the DEAR Collaboration [2] using the the following input parameters: 1) the experimental setup [39] and 2) the theoretical predictions for the hadronic energy level displacements of the  $2p$  state,  $\epsilon_{2p} = -0.6 \text{ meV}$ ,  $\Gamma_{2p} = 2.0 \text{ meV}$ , and the ground state,  $\epsilon_{1s} = 203 \text{ eV}$  and  $\Gamma_{1s} = 226 \text{ eV}$ , of kaonic hydrogen [1].

## 6 Comment on the result

After the manuscript has been posted at archive Faifman and Men'shikov have presented the calculated yields for the  $K$ -series of  $X$ -rays for kaonic hydrogen in dependence of the hydrogen density [41]. They have shown that the use of the theoretical value  $\Gamma_{2p} = 2 \text{ meV}$  of the width of the  $2p$  state of kaonic hydrogen, computed in our work, leads to good agreement with the experimental data, measured for the  $K_\alpha$ -line by the KEK Collaboration [40]. They have also shown that the results of cascade calculations with other values of the width of the  $2p$  excited state of kaonic hydrogen, used as an input parameter, disagree with the available experimental data. The results obtained by Faifman and Men'shikov contradict to those by Jensen and Markushin [13]. Therefore, as has been accentuated by

Faifman and Men'shikov [41], the further analysis of the experimental data by the DEAR Collaboration should allow to perform a more detailed comparison of the theoretical value  $\Gamma_{2p} = 2 \text{ meV}$  with other phenomenological values of the width of the  $2p$  state of kaonic hydrogen  $\Gamma_{2p}$ , used as input parameters.

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